Surface Classification

Jenny Rustad

University of Maryland

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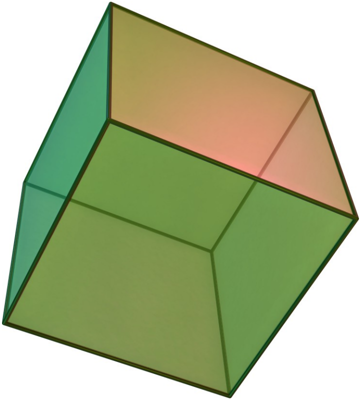
Euler Characteristic = V - E + F



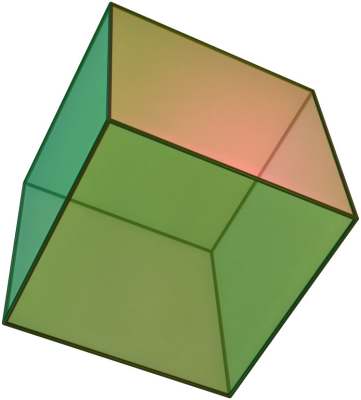
Cube

How many vertices?

How many edges?

How many faces?

Euler Characteristic = V - E + F



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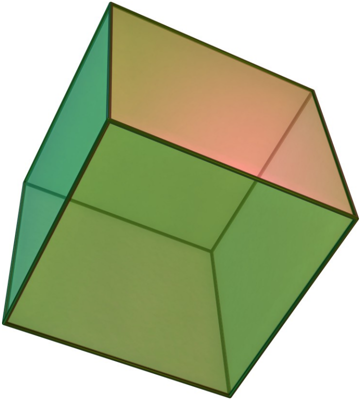

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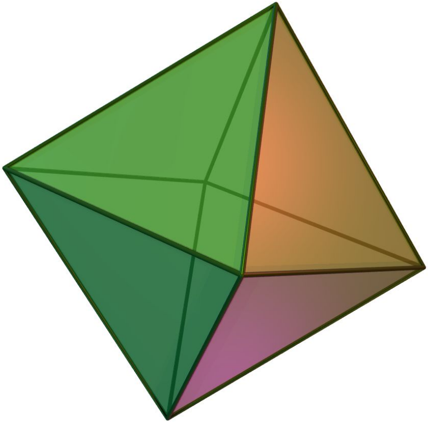
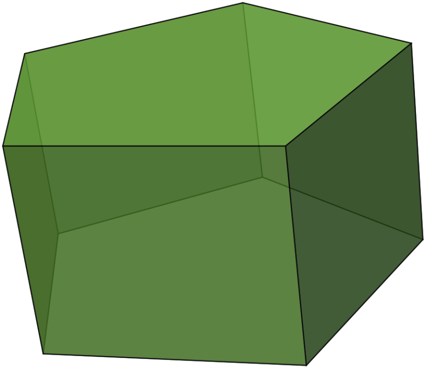
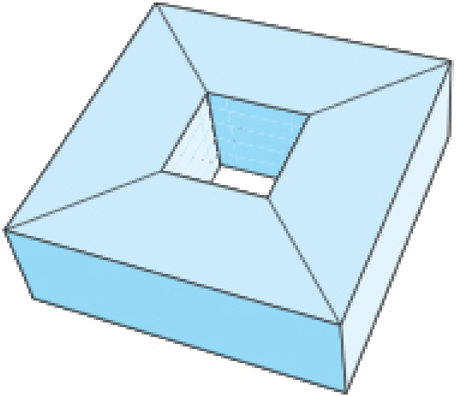


Octahedron



Pentagonal Prism

Euler Characteristic = V - E + F



Topology of Surfaces

Not a big deal:

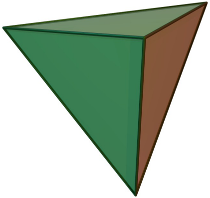
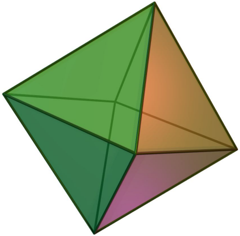
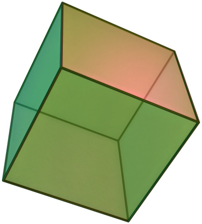
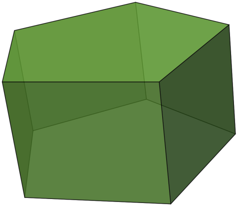
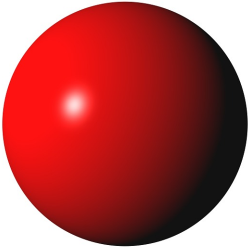
- translating
- rotating
- stretching
- shrinking
- wiggling

Big deal:

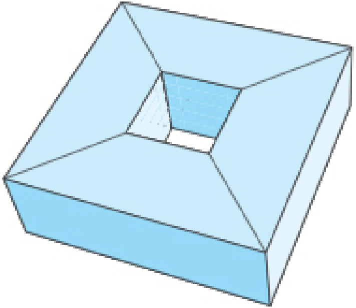
- cutting
- gluing

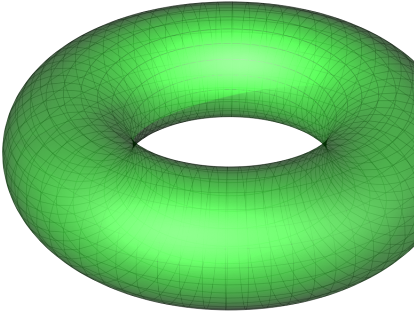
If we can transform one surface into another, using only the kinds of transformations on the left, then these two surfaces are **topologically equivalent** or **homeomorphic**.

Topology of Surfaces

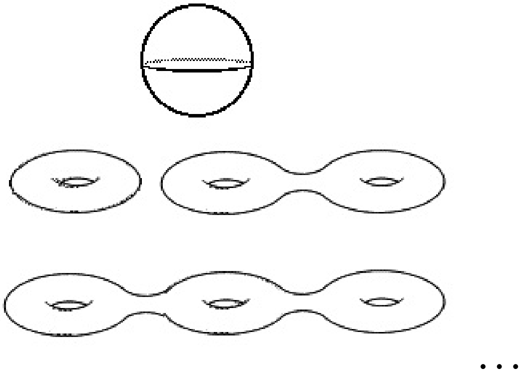
Topology of Surfaces



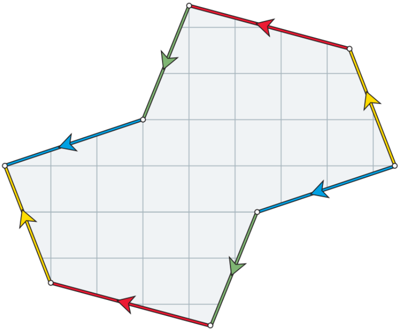


Theorem (Classification of Surfaces)

Let S be a closed, orientable surface. Then S is topologically equivalent to one and only one of the genus g surfaces shown below $(g = 0, 1, 2, \ldots)$:



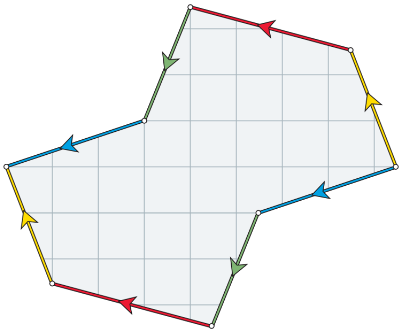
What is that??



This is an example of a translation surface.

But what is it topologically?

What is that??



This is an example of a translation surface.

It's a genus-2 surface!

Further reading (ie, internet search terms)

- Mobius band
- Klein bottle
- Diana Davis math (Prof. Davis is an expert on polygonal billiards who has created many expository materials on the subject for a variety of audiences)